GAS FLOW BETWEEN FIXED CONTACTING SURFACES

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A method is presented for calculating gas flow through the seam between two parts pressed together, with consideration of contact interaction of microroughness on the contacting surfaces of the parts as a function of gas pressure.

One of the important factors significantly affecting the quality of machinery and equipment which uses a gas as an operating substance is the amount of loss through detachable connections (flanges, sleeves, couplings, etc.). These losses are reduced, and sometimes eliminated entirely, by compressing the meeting surfaces of the joint to each other with bolts, dowels, nuts, etc.

Methods for calculation of the loss as a function of the compressive force, properties of the joint materials, geometric characteristics, and roughness parameters are presented in [1, 2]. However, those studies do not consider the displacement of the joint surfaces produced by the pressure of the medium passing through it. And, because of the small size of the gaps through which gas loss occurs (of the order of tenths of a micrometer) even a small displacement of the contacting surfaces significantly affects the amount of loss. This phenomenon is especially marked at high gas pressures, which can reach values of hundreds of MPa in contemporary equipment. At moderate pressures the effect is also significant when the joint contains a layer of highly elastic material, for example, rubber, whose modulus of elasticity is five orders of magnitude less than that of metals [3].

The goal of the present study is to construct a method of calculation of the flow through the seam between joint components with consideration of the components' displacement by gas pressure.

We will use the model proposed in [4], in which the joint consists of a bundle of winding microgaps, formed by the unevenness of the contacting surfaces. Then the gas mass flow rate through the joint can be represented in the form

$$G = \rho_m v_f F. \tag{1}$$

It follows from the equation of state that

$$\rho_m = \frac{p_m \mu}{RT}. \tag{2}$$

According to [5]:

$$v_f = \frac{\Pi v \cos \xi}{\mathrm{Ko}}. \tag{3}$$

In accordance with [4] we write the following expression for the porosity of the joint:

$$\Pi = V/V_c, \tag{4}$$

and write the volume of the free space V and the volume of the joint V_c in the form

$$V = Ah, \quad V_c = AR_{\max}. \tag{5}$$

Combining Eqs. (1)-(5) we will have

$$G = \frac{\mu p_m F h v \cos \xi}{\mathrm{Ko}\, RTR_{\max}}. \tag{6}$$

In the present case the mean velocity v will not be constant over the length of the gap. It increases gradually because the gas pressure falls along the gap length, reducing the displacement of the contacting surfaces, and hence, the height of the gap.

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We will limit ourselves to the case where the gas flowing through the microgap displaces only one of the contacting surfaces; this case is realized, for example, in contact of a rubber gasket with a metallic joint piece. In order to establish the dependence of microgap height h on pressure p, we write the former as the sum of the original height h_0, formed by contact of the opposing surfaces, and an addition Δh, consisting of the displacement under gas pressure of those portions of the surfaces which are not in contact:

$$h(p) = h_0 + \Delta h(p). \tag{7}$$

In defining the original height h_0 we will not consider displacement due to gas pressure. Therefore, the method proposed in [4] may be used to calculate this value, determining the mean height of the gap in the joint as a function of the loading on the joint, the elastic properties of the parts, and the microgeometry of the contacting surfaces. The load, in turn, depends on the clamping force and gas pressure, compensating a portion of the clamping force if there is no self-sealing gasket, or increasing this force if self-sealing occurs, and also on the thickness of the gasket.

In accordance with [4], the expression for the original height h_0 can be written in the form

$$h_0 = R_{\max}(1 - \varepsilon). \tag{8}$$

For elastic contact of the microroughnesses, according to [6], we have

$$\varepsilon = mq^{\beta}. \tag{9}$$

Here

$$m = \left[\frac{2\sqrt{\pi}\, I\Gamma(\nu + 1.5)}{b\Gamma(\nu + 1)}\left(\frac{r}{R_{\max}}\right)^{1/2}\right]^{\beta};$$

$$I = \sum_{i=1}^{2}\frac{1 - \mu_i^2}{E_i}; \quad r = \left(\sum_{i=1}^{2} r_i^{-1}\right)^{-1}; \; \beta = 2/(2\nu + 1).$$

In the most general form the contact pressure q can be represented by

$$q = q_0 + \varphi p_{\text{in}}. \tag{10}$$

Combining Eqs. (8)-(10), we obtain

$$h_0 = R_{\max}[1 - m(q_0 + \varphi p_{\text{in}})^{\beta}]. \tag{11}$$

An analytical expression for the pressure q_0 and the factor φ as functions of the joint parameters can be obtained only by examining the concrete system. The form of these expressions will be determined by: the form of the gasket (plane, toroidal, lens-shaped, etc.); its material (compressible, for example, rubber, or incompressible, for example, steel, indium, etc.; and the presence or absence of self-sealing.

For each combination of the aforementioned attributes, functions can be constructed to describe q_0 and φ by solving the Lamé problem with corresponding boundary conditions.

As an example, we will consider a joint in which a plane ring gasket of incompressible material is installed in a groove of one mating part, with the lateral surface of the gaskets in contact with the inner surface of the groove (Fig. 1), so that self-sealing occurs. We use the solution of the Lamé problem for such a joint [7]:

$$q_0 = \frac{E_1}{3}\left[2\frac{\Delta}{H} + \frac{1}{\alpha^2}\left(2 + \frac{\Delta}{H} - 2\frac{R_s}{R_1}\right)\right], \quad \varphi = 1. \tag{12}$$

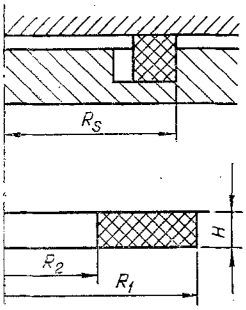
Here $\alpha = R_2/R_1$.

For other construction variants of the joint the expressions for q_0 and φ have a similar character, differing in the consideration of additional factors such as the ring diameter when a toroidal gasket is used, and the fact that in the absence of self-sealing $\varphi < 0$.

Substituting Eq. (12) in Eq. (11), we obtain

$$h_0 = R_{\max}\left\{1 - m\left\{\frac{E_1}{3}\left[2\frac{\Delta}{H} + \frac{1}{\alpha^2}\left(2 + \frac{\Delta}{H} - 2\frac{R_s}{R_1}\right)\right] + p_{\text{in}}\right\}^{\beta}\right\}. \tag{13}$$

As is evident from Eq. (13), the basic height of the microgap h_0 increases with increase in gasket thickness H and other parameters fixed. On the other hand, the value of the additional displacement Δh produced by



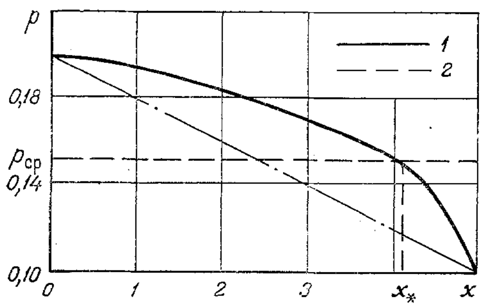


Fig. 1 Fig. 2

Fig. 1. Diagram of joint.

Fig. 2. Pressure distribution over microgap length: 1) calculation with Eq. (17) at $h_0 = 0.5\ \mu$m, $p_{in} = 0.03$ MPa, L = 5 mm, $S_m = 100\ \mu$m, $E_1 = 3.0$ MPa, $\mu_1 = 0.5$; 2) dash-dot linear dependence of pressure on distance along microgap axis. p, MPa; x, mm.

gas pressure is practically independent of gasket thickness, as shown by approximate calculations, following [8]. Therefore, in determining Δh the real gasket can be replaced by a semispace. Using the solution [9] of the problem of displacement of a rectangular portion of a semispace with length equal to the length of the slit $L/\cos\xi$, to simulate the real and microgap, and with width equal to the slit width S_m, under the action of the distributed load, the gas pressure p, we write

$$\Delta h(p) = \varkappa p, \tag{14}$$

where

$$\varkappa = \frac{3n}{E_1}\sqrt{\frac{LS_m}{\cos\xi}}.$$

Substituting Eq. (14) in Eq. (7), we obtain

$$h(p) = h_0 + \varkappa p. \tag{15}$$

Now, when we know the form of the microgap height h as a function of p, we can find how the pressure changes along the gap, using the differential equation [10]

$$\frac{\partial}{\partial x}\left[h^3(p)\frac{\partial p}{\partial x}\right] = 0 \tag{16}$$

with boundary conditions

$$p(0) = p_{in}\ p(L/\cos\xi) = 0.$$

Solving, we have

$$p = \frac{1}{\varkappa}\left\{\left\{(h_0 + \varkappa p_{in})^4 - \frac{x\cos\xi}{L}\left[(h_0 + \varkappa p_{in})^4 - h_0^4\right]\right\}^{\frac{1}{4}} - h_0\right\}. \tag{17}$$

Figure 2 shows the pressure distribution over the microgap length, as calculated with Eq. (17). It is obvious that there is significant deviation from the linearity characteristic of gas flow between contacting surfaces when movement of the surface by gas pressure is negligibly small compared to gap height.

In accordance with [10], we write the following expression for flow velocity:

$$v = \frac{h^2(p)}{12\eta}\frac{\partial p}{\partial x}. \tag{18}$$

After substitution in the right side of Eq. (18) the expressions (15) and (17), we obtain

$$v = \frac{(h_0 + \varkappa p)^2\left[(h_0 + \varkappa p_{in})^4 - h_0^4\right]\cos\xi}{48\eta\varkappa L\left\{(h_0 + \varkappa p_{in})^4 - xL^{-1}\cos\xi\left[(h_0 + \varkappa p_{in})^4 - h_0^4\right]\right\}^{3/4}}. \tag{19}$$

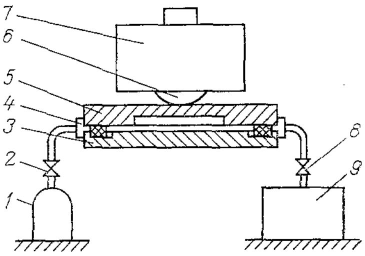


Fig. 3

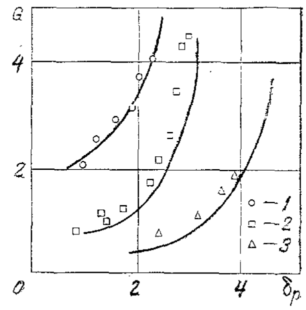


Fig. 4

Fig. 3. Schematic diagram of experimental equipment: 1) reservoir with compressed Freon-12; 2, 8) valves; 3) base; 4) chamber; 5) lid; 6) hemispherical coupling, 7) screw press; 9) GTI-7 flow detector.

Fig. 4. Freon-12 mass flow rate $G\,(10^{-8}$ kg/sec) versus difference of squares of pressures $\delta p\,(10^{8}\,\mathrm{Pa}^2)$ for compression of washer by $\Delta = 0.3$ mm and thicknesses $H = 4$ mm (1), 3 mm (2), 2 mm (3). Curves are calculated with Eq. (22).

As is evident from Eq. (19), the mean gas velocity v changes along the microgap. Since in the expression for flow, Eq. (6), we have the mean pressure p_m, then it is obvious that the velocity v should also be calculated for that section x_*, where the gas pressure reaches the value p_m. Replacing the pressure p on the left side of Eq. (17) with its mean value $p_m = 0.5p_{in} + p_a$, we find

$$x_* = \frac{L}{\cos\xi}\,\frac{(h_0 + \varkappa p_{in})^4 - [h_0 + \varkappa(0.5p_{in} + p_a)]^4}{(h_0 + \varkappa p_{in})^4 - h_0^4}. \tag{20}$$

After substitution of x_* for x and $p_m = 0.5p_{in} + p_a$ for p on the right side of Eq. (19) we obtain

$$v = \frac{[(h_0 + \varkappa p_{in})^4 - h_0^4]\cos\xi}{48\eta\varkappa L\,[h_0 + \varkappa(0.5p_{in} + p_a)]}. \tag{21}$$

Combining Eqs. (6) and (21), we finally obtain

$$G = \frac{\mu F\,(p_{in} + 2p_a)\,[(h_0 + \varkappa p_{in})^4 - h_0^4]\cos^2\xi}{96\,\eta\varkappa LRTR_{max}\mathrm{Ko}}. \tag{22}$$

Equation (22) was verified with experimental apparatus, a diagram of which is shown in Fig. 3. The experiments were performed in the following manner. Freon-12 was supplied to the internal cavity of the joint at a pressure p_{in}, which was varied in steps. At each step the gas loss was determined with a GTI-7 flow detector connected to the chamber enclosing the joint.

Figure 4 shows the Freon-12 mass flow rate as a function of the difference of the squares of the pressures $\delta p = (p_{in} + p_a)^2 - p_a^2$. As is evident from the figure, the dependence of G on δp is significantly nonlinear. This can be explained by deformation of the sealing material under the action of gas pressure.

NOTATION

G, gas mass-flow rate through seam of joint surfaces; ρ_m, mean gas density in seam; v_f, filtration velocity; F, cross-sectional area of seam; p_m, mean gas pressure in seam; μ, molecular weight of gas; R, universal gas constant; T, absolute temperature of gas; Π, porosity of seam; v, mean velocity of gas flow in individual microgap; ξ, mathematical expectancy of angle of inclination of microgap axis to the normal to the faces of the parts forming the joint (in the case where the axis distribution over the range $(0, \pi/2)$ is equiprobable, it can be assumed that $\xi = \pi/4$); Ko, empirical coefficient (Koseni–Karman constant); V, volume of free space within joint; V_c, volume of joint; h_0, basic height of microgap formed by contact of opposing joint surfaces; Δh, addition to basic height due to displacement under gas pressure of noncontacting joint surface areas; R_{max}, maximum height of roughness on contacting surfaces; ε, relative approach of mating surfaces due to load on joint; $\Gamma(z)$, gamma function of variable z; b, ν, parameters of reference curve of net mating surfaces; E_i, μ_i, modulus of elasticity and Poisson coefficient of material in i-th part of joint; r_i, mean radius of curvature of microroughness of i-th mating surface; q, contact pressure on mating surface due to simultaneous action of external compression and gas pressure; q_0, contact pressure due to external compression; φ, factor dependent on construction characteristics of joint and Poisson coefficient; p_{in}, excess gas pressure in internal joint cavity; E_1, modulus of elasticity of gasket material; Δ, specified compression of gasket in the joint; R_s, larger radius of groove in which gasket is installed; R_1 and R_2, outer and inner radii of gasket before installation in joint; L, width of gasket; S_m, mean distance between adjacent microroughness peaks of mating surfaces; n, coefficient dependent on ratio L/S_m (tabulated in [9]); x, coordinate measured from inner face of gasket along microgap axis; p, current gas pressure in microgap; x_*, microgap section in which gas pressure p reaches its mean value p_m.

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FLOW AND HEAT- AND MASS-TRANSFER IN A THIN FILM ON A FLUTED SURFACE

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A study is made of steady flows and heat- or mass-transfer in a thin liquid film flowing over a sloping fluted surface in a direction perpendicular to its generatrix.

Most theoretical studies of hydrodynamics and transfer processes in thin liquid films investigate films on flat substrates (see the survey in [1, 2], for example.). In practice, various types of fluted surfaces and surfaces with ragging, threads, or another type of artificial roughness are often used to intensify heat- and mass-transfer. Examples of theoretical analysis of films on such surfaces may be found in [3-7].

Works on this subject, including [3-7], usually contain a significant number of inaccuracies and invalid assumptions. The main purpose of the present work is therefore to explain, in a detailed and rigorous manner, a small-parameter method which can be successfully used to solve problems of this type. This is done using the example of steady two-dimensional flow of a film over a fluted substrate with a horizontal generatrix.

Formulation of the Problem

Let the middle plane of the fluted surface form an angle α with the vertical. We will introduce Cartesian coordinate axes ξ and η, oriented, respectively, in the direction of the projection **g** onto this plane coincident with the direction of motion, and normal to the plane. We will describe the surface using the periodic function

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